Game Theory

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Zur Theorie der Gesellschaftsspiele[1]).

Von

J. v. Neumann in Berlin.

Einleitung.

**1.** Die Frage, deren Beantwortung die vorliegende Arbeit anstrebt, ist die folgende:

n Spieler, S_1, S_2, ..., S_n, spielen ein gegebenes Gesellschaftsspiel $\mathfrak{G}$. Wie muß einer dieser Spieler, S_m, spielen, um dabei ein möglichst günstiges Resultat zu erzielen?

Die Fragestellung ist allgemein bekannt, und es gibt wohl kaum eine Frage des täglichen Lebens, in die dieses Problem nicht hineinspielte; trotzdem ist der Sinn dieser Frage kein eindeutig klarer. Denn sobald $n > 1$ ist (d. h. ein eigentliches Spiel vorliegt), hängt das Schicksal eines jeden Spielers außer von seinen eigenen Handlungen auch noch von denen seiner Mitspieler ab; und deren Benehmen ist von genau denselben egoistischen Motiven beherrscht, die wir beim ersten Spieler bestimmen möchten. Man fühlt, daß ein gewisser Zirkel im Wesen der Sache liegt.

Wir müssen also versuchen, zu einer klaren Fragestellung zu kommen. Was ist zunächst ein Gesellschaftsspiel? Es fallen unter diesen Begriff sehr viele, recht verschiedenartige Dinge: von der Roulette bis zum Schach, vom Bakkarat bis zum Bridge liegen ganz verschiedene Varianten des Sammelbegriffes „Gesellschaftsspiel" vor. Und letzten Endes kann auch irgendein Ereignis, mit gegebenen äußeren Bedingungen und gegebenen Handelnden (den absolut freien Willen der letzteren vorausgesetzt), als Gesellschaftsspiel angesehen werden, wenn man seine Rückwirkungen auf die in ihm handelnden Personen betrachtet[2]). Was ist nun das gemeinsame Merkmal aller dieser Dinge?

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[1]) Der Inhalt dieser Arbeit ist (mit einigen Kürzungen) am 7. XII. 1926 der Göttinger Math. Ges. vorgetragen worden.
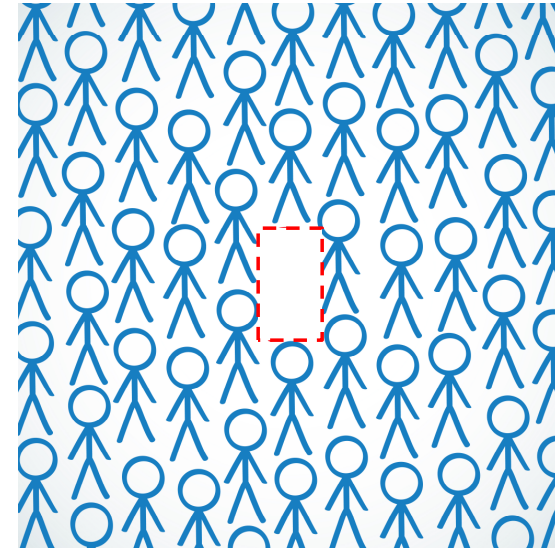
[2]) Es ist das Hauptproblem der klassischen Nationalökonomie: was wird, unter gegebenen äußeren Umständen, der absolut egoistische „homo œconomicus" tun?
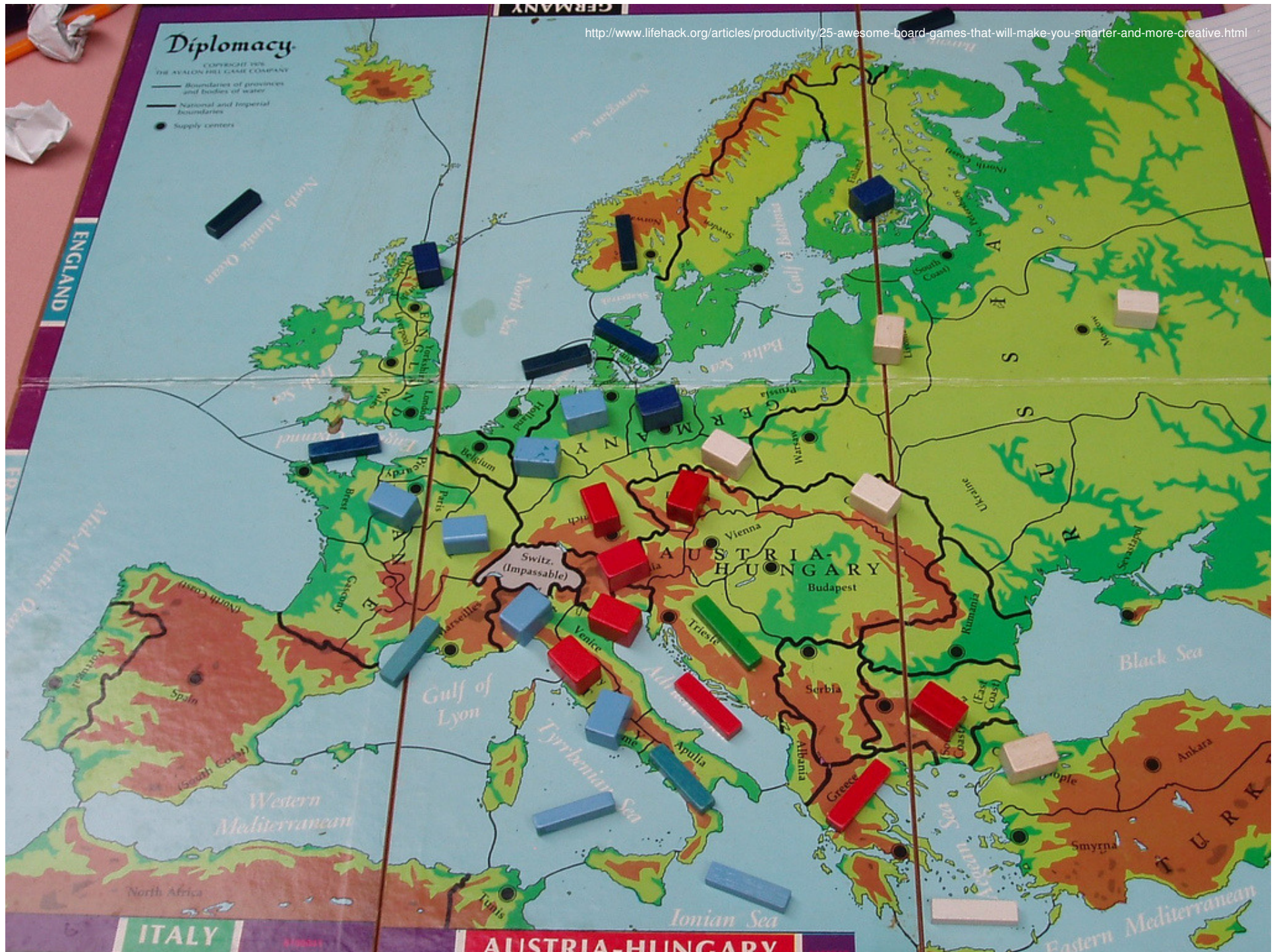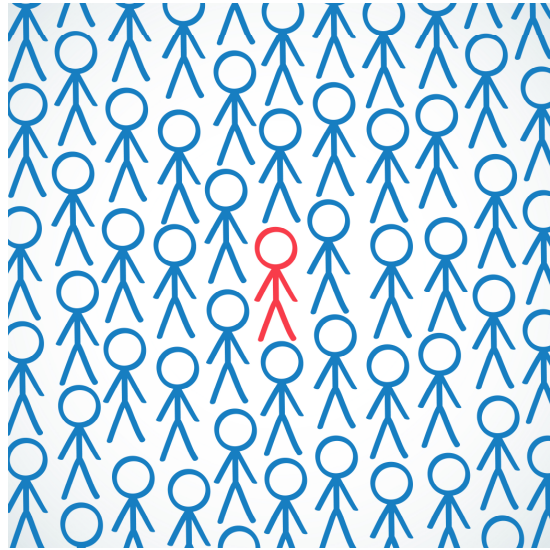
The "value" created
by these players

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The "value" created
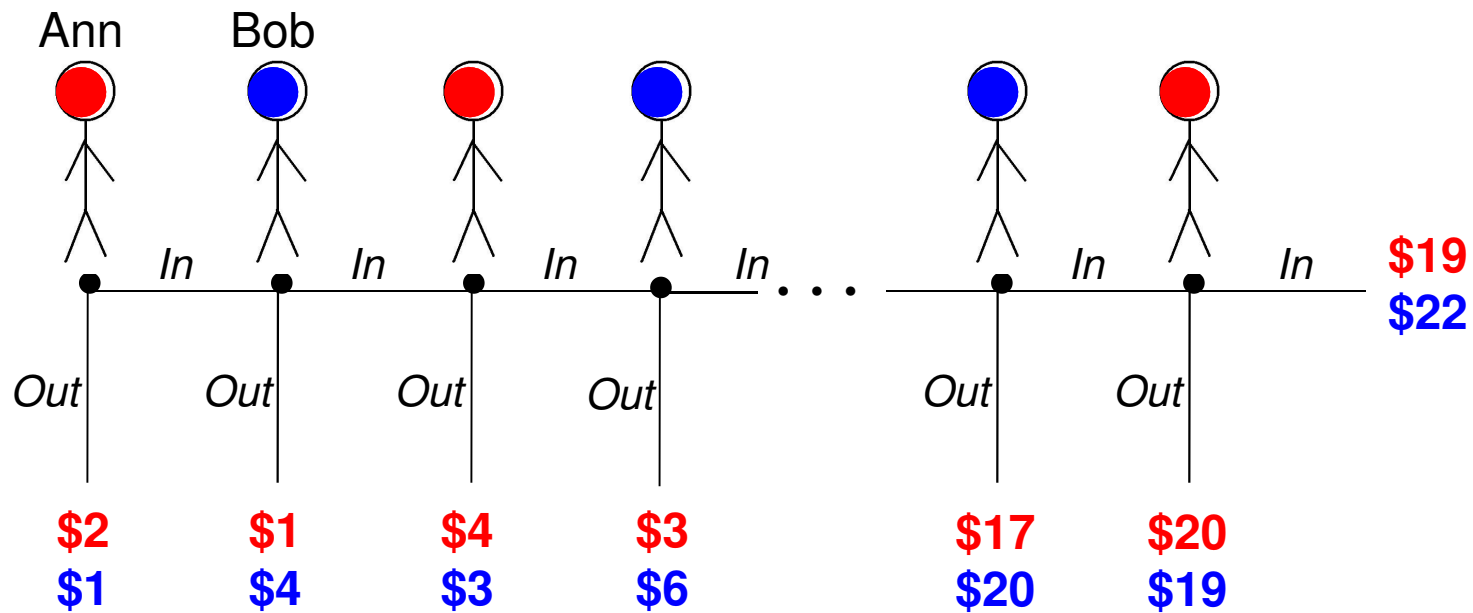by these players

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The "added value"
of this player

John von Neumann: "Each player must choose his strategy in 'complete ignorance'"
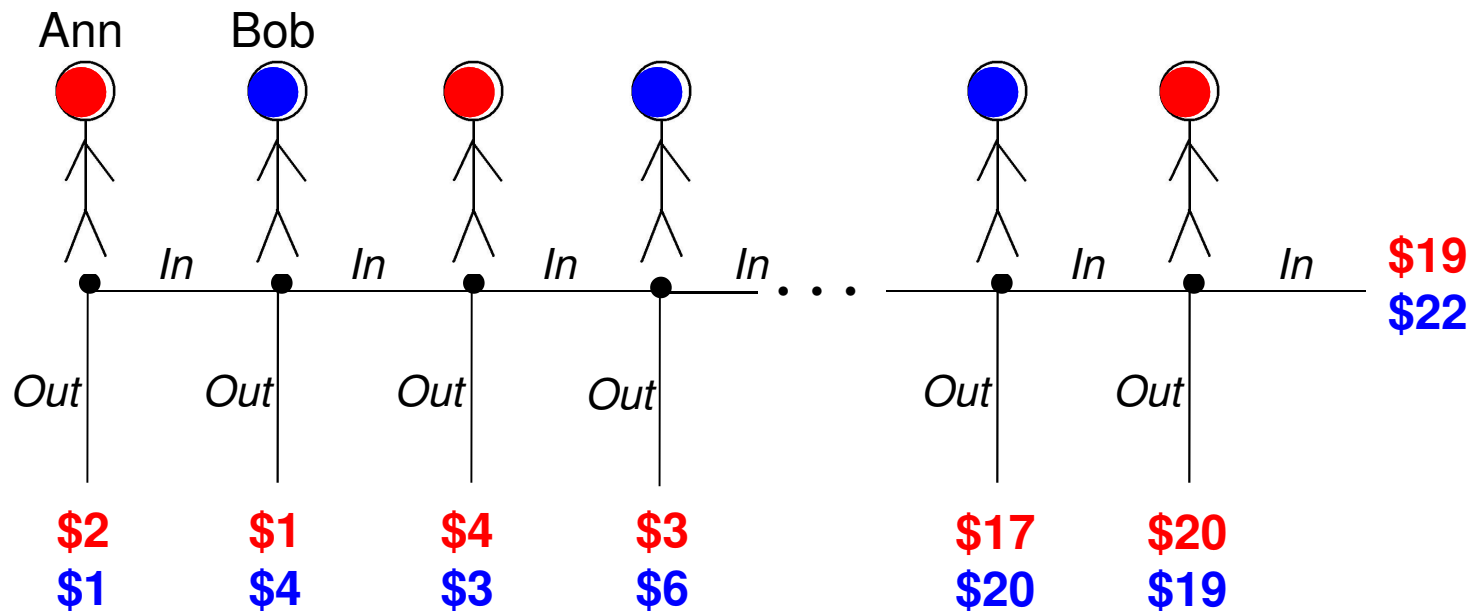
John Nash: "[A] rational prediction should be unique"

"In game theory, the Nash equilibrium is a solution concept of a non-cooperative game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy."

The Centipede Game
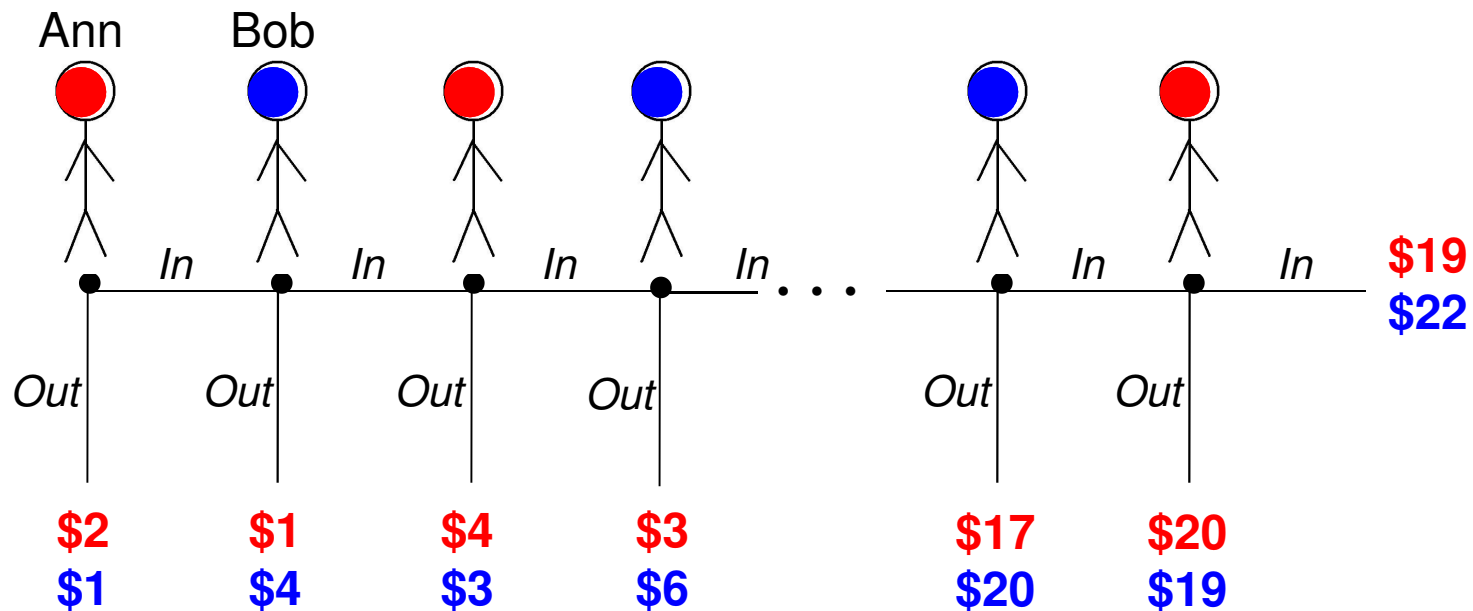
Rosenthal, R., "Games of Perfect Information, Predatory Pricing, and the Chain Store," *Journal of Economic Theory*, 1980, 25, 92-100

McKelvey, R., and T. Palfrey, "An Experimental Study of the Centipede Game," 1992, *Econometrica*, 60, 803-836

The Centipede Game

In any Nash equilibrium of the Centipede game, Ann chooses *Out* immediately

Rosenthal, R., "Games of Perfect Information, Predatory Pricing, and the Chain Store," *Journal of Economic Theory*, 1980, 25, 92-100
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http://commons.wikimedia.org/wiki/File:Stick_Figure.jpg

The Centipede Game

In any Nash equilibrium of the Centipede game, Ann chooses *Out* immediately

This prediction is consistently falsified in experiments

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McKelvey, R., and T. Palfrey, "An Experimental Study of the Centipede Game," 1992, *Econometrica*, 60, 803-836
http://commons.wikimedia.org/wiki/File:Stick_Figure.jpg

http://www.fanpop.com/clubs/sherlock/images/32478689/title/sherlock-john-wallpaper

THE LANGUAGE OF GAME THEORY

Putting Epistemics into the Mathematics of Games

Adam Brandenburger

Foreword by Eric S. Maskin
Series Editor-in-Chief

Vol. 5

THE LANGUAGE OF GAME THEORY

Brandenburger

"Adam Brandenburger's work on the knowledge requirements implicit in game theory has become classic. These are of profound importance in understanding the relevance of game theory and, indeed, economic theory in general to the real economy. It is very good to have them collected, with an introduction that brings out the underlying themes."

Kenneth J. Arrow
Stanford University, USA

"Three hundred years ago, Francis Waldegrave found the first minimax solution of a matrix game. But in his correspondence with mathematicians Pierre Rémond de Montmort and Nicolaus Bernoulli, Waldegrave counseled that epistemic considerations involving knowledge, beliefs, uncertainty, and incomplete information also mattered. The principal practitioners of game theory, with the notable exceptions of John Harsanyi and Robert Aumann, have ignored this advice. In recent years, these two theorists have been joined by Adam Brandenburger, whose work on epistemic game theory has been collected in this splendid volume. Eight classic papers by Brandenburger with a number of co-authors present an authoritative view of the field while an insightful introduction provides a roadmap to research both present and future."

Harold W. Kuhn
Princeton University, USA

"This book features a collection of foundational papers by Adam Brandenburger in epistemic game theory. Though still evolving, this approach marks a tectonic shift in game theory by offering a new, epistemic dimension which might be compared to the introduction of synchronized sound to motion pictures in the early 20th century: it might not immediately provide a complete picture, but it has the potential of changing the field forever."

Sergei N. Artemov
The Graduate Center of the City University of New York, USA

This volume contains eight papers written by Adam Brandenburger and his co-authors over a period of 25 years. These papers are part of a program to reconstruct game theory in order to make how players reason about a game a central feature of the theory. The program — now called epistemic game theory — extends the classical definition of a game model to include not only the game matrix or game tree, but also a description of how the players reason about one another (including their reasoning about other players' reasoning). With this richer mathematical framework, it becomes possible to determine the implications of how players reason for how a game is played. Epistemic game theory includes traditional equilibrium-based theory as a special case, but allows for a wide range of non-equilibrium behavior.

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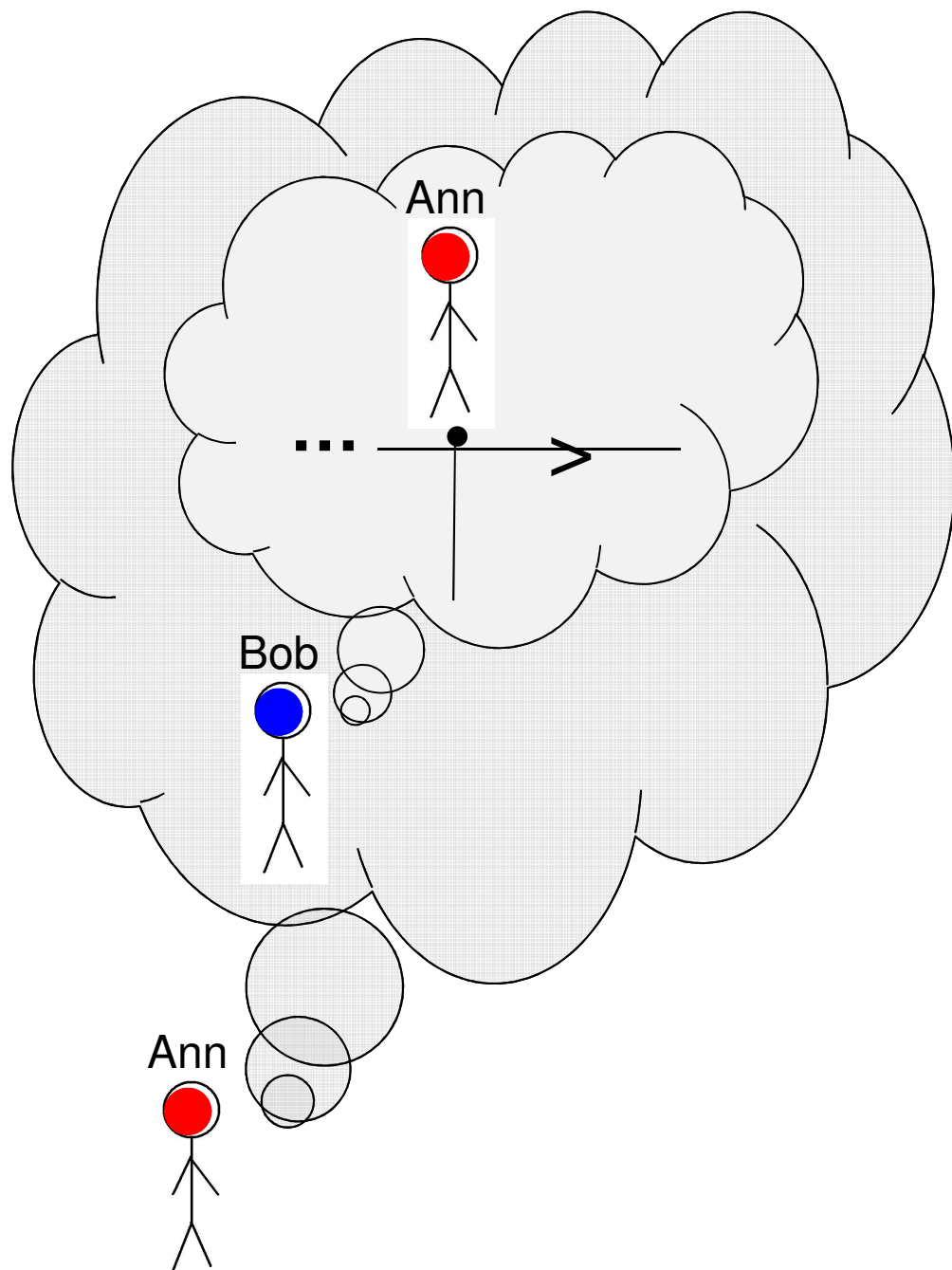
Mathematics of probabilities over

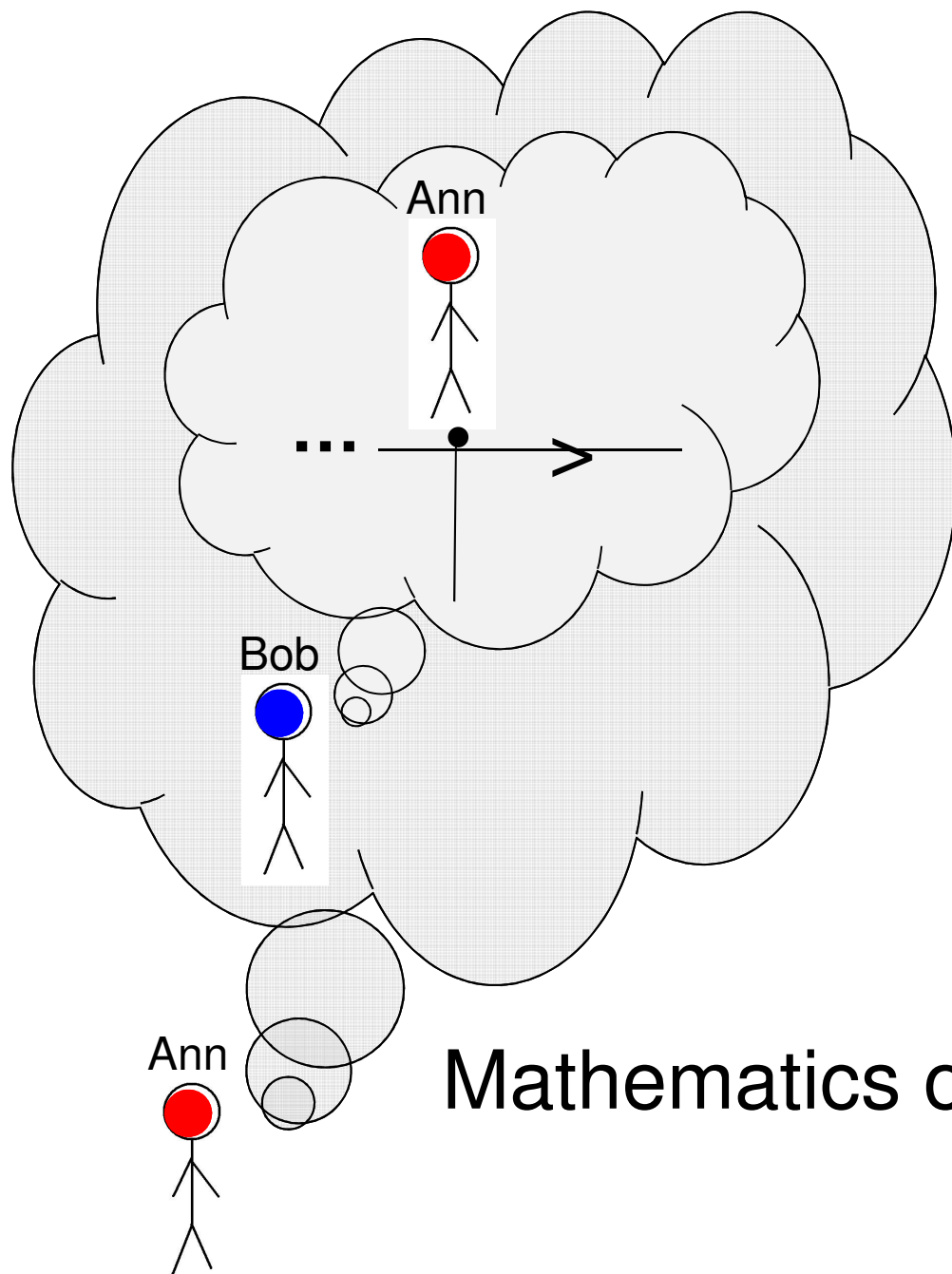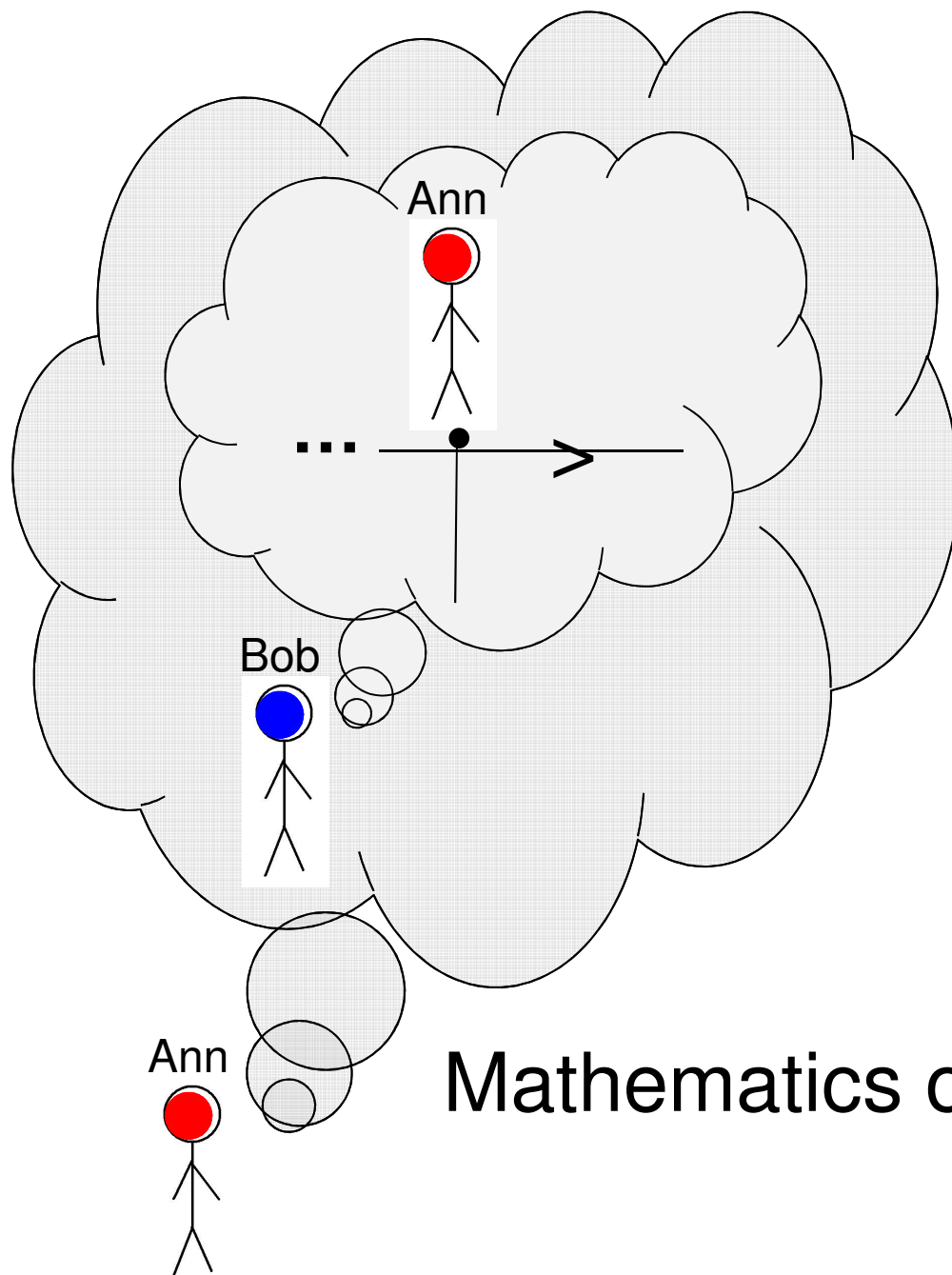probabilities over …

Mathematics of probabilities over

Ann thinks that

Bob thinks that

Ann thinks that what Bob thinks is wrong

Brandenburger, A., and H. Jerome Keisler, "An Impossibility Theorem on Beliefs in Games," *Studia Logica*, 84, 2006, 211-240
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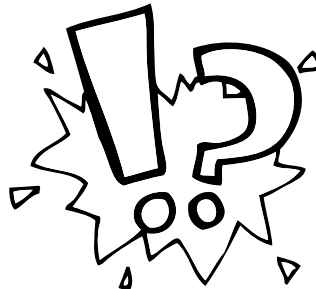
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Does:

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Ann *not* think that what Bob thinks is wrong?

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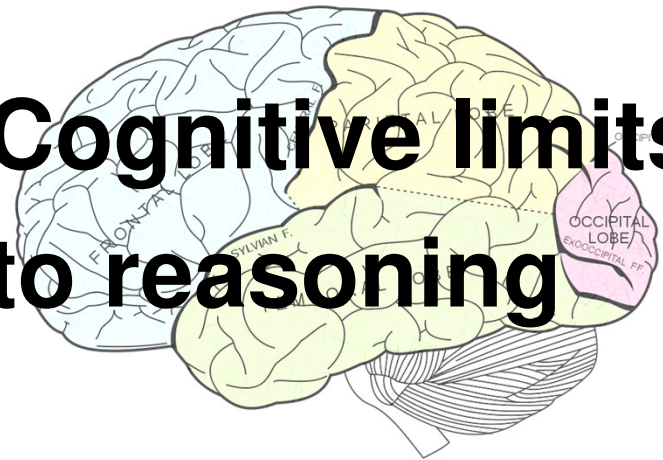Ann *not* think that what Bob thinks is wrong?

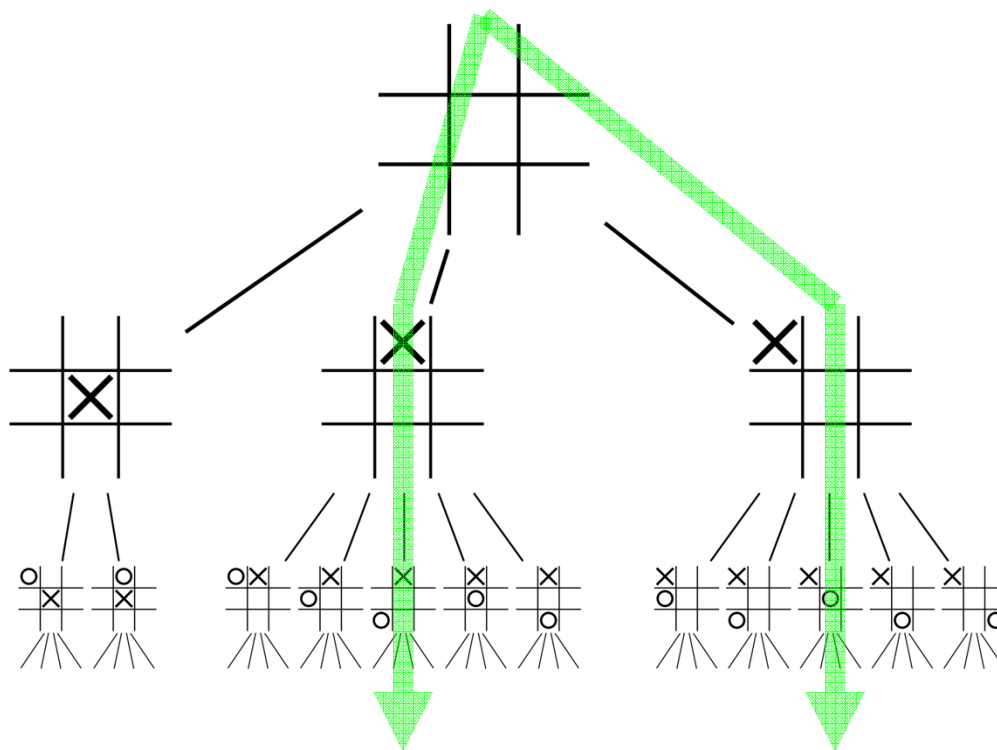Brandenburger, A., and H. Jerome Keisler, "An Impossibility Theorem on Beliefs in Games," *Studia Logica*, 84, 2006, 211-240
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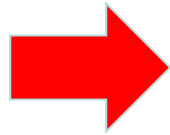Ann *not* think that what Bob thinks is wrong?

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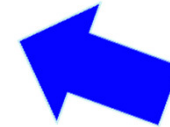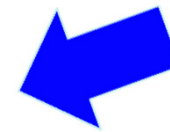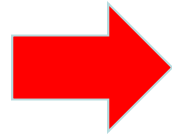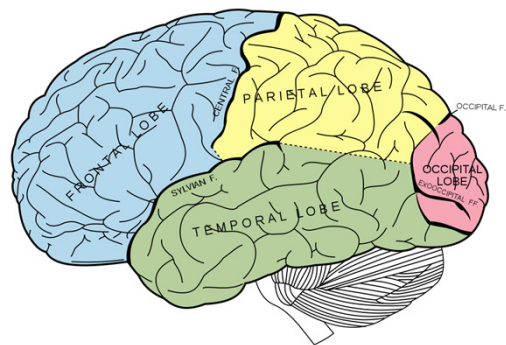
Theoretical limits to reasoning → **Cognitive limits to reasoning**
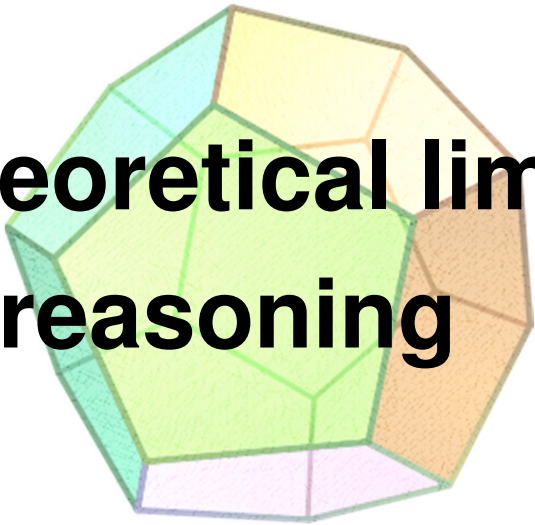
Build theory to infer (bounds on) the number of levels of reasoning from observing paths of play

Brandenburger, A., and A. Friedenberg, "How Many Levels Do Players Reason? An Observational Challenge and Solution," October 2014, at adambrandenburger.com/articles/papers
http://en.wikipedia.org/wiki/Game_tree#mediaviewer/File:Tic-tac-toe-game-tree.svg

Your move …
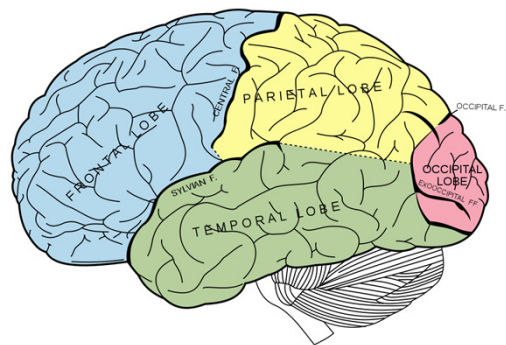
or

McCabe, K. et al., "A Functional Imaging Study of Cooperation in Two-Person Reciprocal Exchange," *PNAS*, 98, 2001, 11832-11835
Gallagher, H., et al., "Imaging the Intentional Stance in a Competitive Game," *NeuroImage*, 16, 2002, 814-821
Rilling, J., et al., "The Neural Correlates of Theory of Mind Within Interpersonal Interactions," *NeuroImage*, 22, 2004, 1694-1703
http://upload.wikimedia.org/wikipedia/commons/thumb/1/1a/Gray728.svg/1024px-Gray728.svg.png

Sherlock Holmes on his brother Mycroft Holmes: "And yet he was absolutely incapable of working out the practical points …"